Test der Erzeugung der Feinstrukturkonstante

1. Stoßtransformationen

Relativgeschwindigkeit:

$$W(u,v) := v - u \qquad \text{z.B.} \qquad u := \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad v := \begin{pmatrix} -2 \\ 0.7 \\ 0.3 \end{pmatrix} \tag{1}$$

$$W(u,v) = \begin{pmatrix} -3 \\ -0.3 \\ -0.7 \end{pmatrix}$$

Richtung der Relativgeschwindigkeit:

$$\Phi(u,v) := \text{if } W(u,v)_1 > 0 \quad \text{atan}\left(\frac{W(u,v)_2}{W(u,v)_1}\right) \quad \text{else if } W(u,v)_1 = 0 \quad \text{sign}\left(W(u,v)_2\right)\cdot\frac{\pi}{2} \quad \text{else if } \left(W(u,v)_1 < 0\right) \wedge \left(W(u,v)_2 \geq 0\right) \quad \text{atan}\left(\frac{W(u,v)_2}{W(u,v)_1}\right) + \pi \quad \text{else} \quad \text{atan}\left(\frac{W(u,v)_2}{W(u,v)_1}\right) - \pi \tag{2}$$

$$\Phi(u,v) = -3.0419$$

$$\Theta(u,v) := \text{acos}\left(\frac{W(u,v)_3}{\sqrt{W(u,v)_1^2 + W(u,v)_2^2 + W(u,v)_3^2}}\right) \tag{3}$$

$$\Theta(u,v) = 1.7989$$

$$S_z(\theta_s, \varphi_s) := \begin{pmatrix} \sin(\theta_s)\cdot\cos(\varphi_s) \\ \sin(\theta_s)\cdot\sin(\varphi_s) \\ \cos(\theta_s) \end{pmatrix} \quad (4)$$

$$Dz(u, v) := \begin{pmatrix} \cos(\Phi(u, v)) & \sin(\Phi(u, v)) & 0 \\ -\sin(\Phi(u, v)) & \cos(\Phi(u, v)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$$Dy(u, v) := \begin{pmatrix} \cos(\Theta(u, v)) & 0 & -\sin(\Theta(u, v)) \\ 0 & 1 & 0 \\ \sin(\Theta(u, v)) & 0 & \cos(\Theta(u, v)) \end{pmatrix} \quad (6)$$

Stoßachse:

$$S(u, v, \theta_s, \varphi_s) := Dz(u, v)^{-1}\cdot Dy(u, v)^{-1}\cdot S_z(\theta_s, \varphi_s) \quad (7)$$

$$u_p(u, v, \theta_s, \varphi_s) := S(u, v, \theta_s, \varphi_s)\cdot\left(S(u, v, \theta_s, \varphi_s)\cdot u\right) \quad (8)$$

$$v_p(u, v, \theta_s, \varphi_s) := S(u, v, \theta_s, \varphi_s)\cdot\left(S(u, v, \theta_s, \varphi_s)\cdot v\right) \quad (9)$$

$$u_o(u, v, \theta_s, \varphi_s) := u - u_p(u, v, \theta_s, \varphi_s) \quad (10)$$

$$v_o(u, v, \theta_s, \varphi_s) := v - v_p(u, v, \theta_s, \varphi_s) \quad (11)$$

Stoßtransformationen:

$$u_s(u, v, \theta_s, \varphi_s) := v_p(u, v, \theta_s, \varphi_s) + u_o(u, v, \theta_s, \varphi_s) \quad (12)$$

$$v_s(u, v, \theta_s, \varphi_s) := u_p(u, v, \theta_s, \varphi_s) + v_o(u, v, \theta_s, \varphi_s) \quad (13)$$

Beispielsweise bei Frontalstoß:

$$u_s\left(\begin{pmatrix}0\\0\\\frac{1}{2}\end{pmatrix}, \begin{pmatrix}0\\0\\-1.5\end{pmatrix}, \frac{\pi}{4}, \frac{\pi}{2}\right) = \begin{pmatrix}0\\1\\-0.5\end{pmatrix}$$

$$v_s\left(\begin{pmatrix}0\\0\\\frac{1}{2}\end{pmatrix}, \begin{pmatrix}0\\0\\-1.5\end{pmatrix}, \frac{\pi}{4}, \frac{\pi}{2}\right) = \begin{pmatrix}0\\-1\\-0.5\end{pmatrix}$$